

# Ab Initio Liquid Hydrogen Muon Cooling Simulations with ELMS in ICOOL

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**Abstract.** This paper presents new theoretical results on the passage of muons through liquid hydrogen which have been confirmed in a recent experiment. These are used to demonstrate that muon bunches may be compressed by ionisation cooling more effectively than suggested by previous calculations.

Muon cooling depends on the differential cross section for energy loss and scattering of muons. We have calculated this cross section for liquid  $H_2$  from first principles and atomic data, avoiding traditional assumptions. Thence, 2-D probability maps of energy loss and scattering in mm-scale thicknesses are derived by folding, and stored in a database. Large first-order correlations between energy loss and scattering are found for  $H_2$ , which are absent in other simulations. This code is named ELMS, Energy Loss & Multiple Scattering. Single particle trajectories may then be tracked by Monte Carlo sampling from this database on a scale of 1 mm or less. This processor has been inserted into the cooling code ICOOL. Significant improvements in 6-D muon cooling are predicted compared with previous predictions based on GEANT. This is examined in various geometries. The large correlation effect is found to have only a small effect on cooling. The experimental scattering observed for liquid  $H_2$  in the MUSCAT experiment has recently been reported to be in good agreement with the ELMS prediction, but in poor agreement with GEANT simulation.

## 1. Introduction

New particle accelerators may be built that are based on intense beams of high energy muons. Such beams would provide a source of highly collimated neutrinos, and later muon colliding beams [1, 2]. To achieve such beams the muons must be compressed and cooled to increase the phase space density. It has been pointed out that passage of the beam through low  $Z$  material, followed by RF acceleration to replace the longitudinal momentum lost, would cool the beam transversely while suffering the least heating due to scattering. A rigorous understanding of the passage of muons through liquid hydrogen is therefore required. The longitudinal phase space dynamics of the muons depends on both the muon velocity and the change in velocity; a thorough understanding and accurate simulation is therefore required in this case also.<sup>‡</sup>

<sup>‡</sup> A fuller exposition of the theory and simulation will be published later.

## 2. Simulation process

### 2.1. Cross section

Each collision of a muon in a material involves both energy transfer and momentum transfer. For example to predict reliably the cooling and heating effects of the passage of a muon bunch with energies  $E$  through a thickness of liquid hydrogen requires a knowledge of the appropriate cross section for energy transfer  $\mathcal{E}$  and transverse momentum transfer  $p_t$ ,

$$\frac{d^2\sigma}{d\mathcal{E}dp_t}(E, \mathcal{E}, p_t).$$

This cross section is determined uniquely by:

- the known  $\mu$ -proton cross section§ with  $Q^2$  dependence|| modified by the screening of the proton charge by the known electron atomic wavefunction;
- the excitation and ionisation cross section of molecular hydrogen that can be deduced [3] from the known photoabsorption spectrum of  $H_2$  [4];
- the  $\mu$ -electron cross section for  $Q^2$  greater than that involved in photoelectric atomic ionisation.¶

As discussed elsewhere [3, 5, 8], these parts do not interfere and parts 2 & 3 are joined by the Bethe sumrule [3]. The  $H_2$  photoabsorption spectrum requires a small correction for density. This is well determined by the measured refractive index of liquid  $H_2$ . The only uncertainty is the role played by higher multipoles at high  $Q^2$ . Reasonable modifications suggest that this effect is everywhere less than 2% [8].

The cross section, simplified in standard treatments into separate energy loss and scattering processes, involves correlations between energy loss and scattering. This is important in hydrogen where scattering as well as energy loss through collisions with atomic electrons is significant.

The cross section includes the full range of excitation, ionisation, multiple scattering, relativistic recoil, Cherenkov effect, delta rays, nuclear and atomic form factors in a unified way. Bremsstrahlung contributions have been calculated but are insignificant here.

### 2.2. Folding probabilities for thin-absorber database

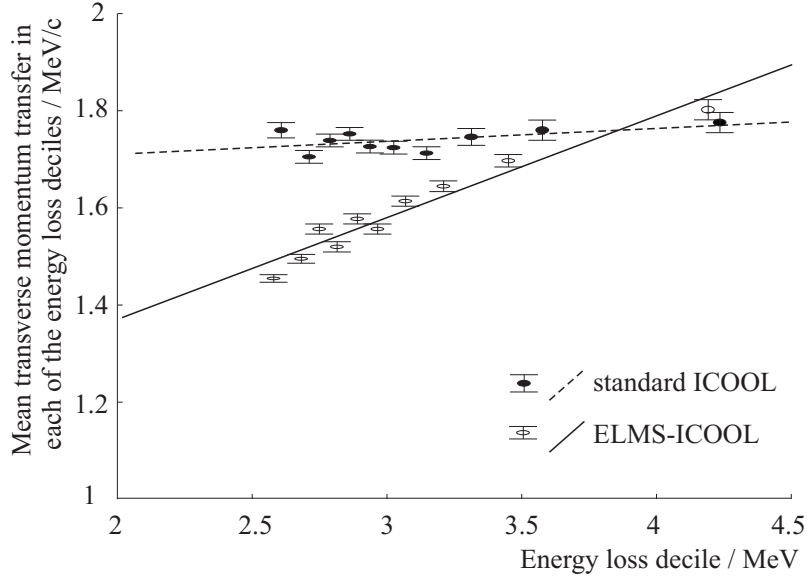
The collision frequency in liquid  $H_2$  is several million per m, while some significant collisions are rare. This relates to the  $1/Q^4$  of the Rutherford cross section and the large range of  $Q^2$  available. Therefore tracking particles by Monte Carlo at the microscopic level is statistically poorly behaved.

A different approach is to use the cross section to tabulate 2-D probability maps of energy and scattering by folding the maps from thinner samples. Thus we have constructed such maps for thicknesses of a mm or less for 138 momenta on a logarithmic scale from 5 to 53000 MeV/c, close enough to permit interpolation to 1% accuracy. The maps are on a 5% logarithmic grid in energy loss and  $p_t$ . In folding together maps of thinner samples the azimuth of the scattering in the two layers is stepped through.

§ This is the Rosenbluth modification of the Rutherford cross section.

|| The variable  $Q^2$  is the 4-momentum transfer squared,  $p^2 - (\mathcal{E}/c)^2$ , where  $p$  is the 3-momentum transfer.

¶ This is the Dirac modification of the Rutherford cross section.



**Figure 1.** Correlation between scattering and energy loss at 200MeV/c in 10cm H<sub>2</sub> as simulated by standard ICOOL and by ELMS-ICOOL. The mean transverse momentum transfer is plotted for each energy loss decile.

The layers have to be thin enough that it may be assumed that the cross section and the path traversed within the layer are not changed as a result of collisions within the layer. The result is a database of normalised probability maps which may be sampled.

Derived secondary formulae such as Molière scattering, Bethe–Bloch mean energy loss, multiple scattering from the radiation length and other short cuts that make Gaussian assumptions are not used. The calculations are not tuned as there are no free parameters.

### 2.3. MC sampling of thin absorber database for general cooling studies

For absorbers of arbitrary thickness the database may be sampled and individual muons tracked taking full account of changes in direction and momentum, following each step. The mm-scale step length is decreased with  $\beta^2$  and is affected by interpolation. Tracking is fast.

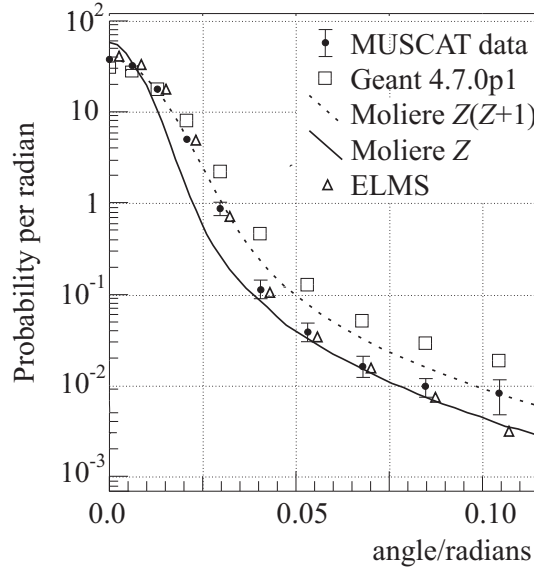
The ELMS sampling algorithm with its database has been embedded in the ICOOL [9] muon tracking code, taking care to maintain step-length compatibility.<sup>+</sup>

## 3. Results

### 3.1. Correlation effects

Fig. 1 shows that correlation between scattering and energy loss persists even in finite absorber thicknesses. A second order effect whereby a muon that loses a large energy then becomes more liable to suffer large scattering is evident in the standard

<sup>+</sup> Standard ICOOL uses algorithms very close to those of GEANT3. For materials other than hydrogen, e.g. absorber windows, ELMS-ICOOL uses standard energy loss and scattering algorithms.



**Figure 2.** Experimental distribution in multiple scattering in 15.9cm of liquid H<sub>2</sub> from the MUSCAT collaboration [11], compared with various calculations including ELMS. The data are shown with error bars where these are significant. The ELMS points are all displaced to the right of the data points for clarity.

ICOOL simulation. However the first order effect due to the differential cross section included in ELMS-ICOOL is much larger. We have studied the effect of suppressing this correlation in ELMS-ICOOL simulations, and found it has no effect on cooling characteristics [8].

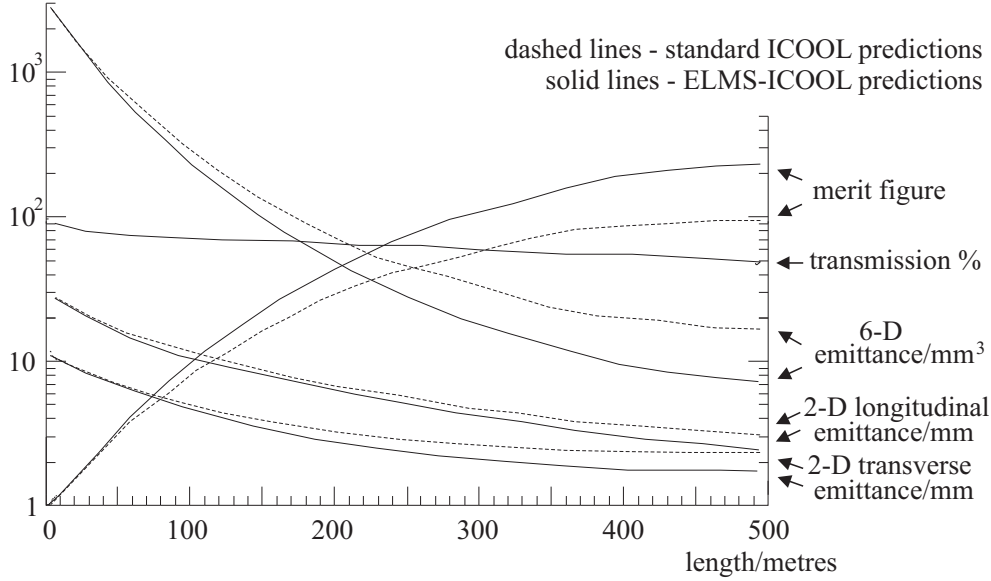
### 3.2. Comparison with the MUSCAT experiment at 172MeV/c

Multiple scattering distributions have been compared with the recent observations of the MUSCAT collaboration [11]. These data corrected for the effect of windows are shown for 15.9cm of liquid H<sub>2</sub> in Fig. 2. The agreement with ELMS is remarkable. The Molière calculation when it includes electron scattering fits the data at small angles while at large angles the limited  $\mu$ -e centre-of-mass momentum prevents such a contribution. This was pointed out by Tollestrup [13]. Such kinematic effects are naturally included in ELMS. GEANT [10] gives a poor description.

For cooling the energy loss is as important as the scattering distribution. However there is less difference between calculations. For example, the mean range as a function of momentum determined by ELMS agrees with tabulated values of range [8]. This is a non-trivial check.

We may conclude that ELMS is a good basis on which to predict the performance of a muon cooler.

Similar calculations for other materials have not yet been made. However differences are not expected to be so marked since the combination of energy loss and scattering from atomic electrons is less significant for greater values of  $Z$ .



**Figure 3.** Summary plot of the cooling performance with distance of an RFOFO ring.

### 3.3. Simulation of MICE at 200MeV/c

We have considered the cooling in transverse emittance by a repeated MICE\* channel 100m long including the effect of windows. Simulations with standard ICOOL and with ELMS-ICOOL showed that 90% of the beam survived, equilibrium emittance was not reached, and transverse cooling of 55% (standard ICOOL) and 59% (ELMS-ICOOL) were predicted.

### 3.4. Simulation of RFOFO ring at 200MeV/c

The cooling in 6-D phase space has been studied in the context of the RFOFO cooling ring [12]. This offers a way to achieve simultaneous longitudinal and transverse cooling by using wedge shaped absorbers in a circular ring of absorbers and RF cavities. The results are shown in Fig. 3 for standard ICOOL and ELMS-ICOOL. The cooling in transverse and normalised longitudinal emittances are combined to give the 6-D emittance. This may be used with the transmission efficiency to give an overall Merit Factor, defined as the increase in central 6-D phase space density including losses in transmission. While the transmission is the same (51% & 49%) in 500m (15 turns), ELMS predicts an increase in the Merit Factor to 230 from the value of 100 found by standard ICOOL. If the bunch charge is integrated over a finite central volume the cooling factor falls. However the improvement predicted by ELMS persists [8].

Instead of simulating the progressive cooling of muons towards an asymptotic normalised equilibrium emittance, the equilibrium emittance itself may be studied. Samples of  $10^5$  muons with  $\beta_t = 40\text{cm}$  were considered with different values of incident transverse emittance. The value at which the normalised emittance following transport through 20cm of liquid  $\text{H}_2$  did not change was noted. This equilibrium

\* Muon Ionisation Cooling Experiment [7, 8]

transverse emittance was found to be 1.60mm by standard ICOOL but 1.18mm by ELMS-ICOOL.

#### 4. Conclusion

We have pointed out that the cross section of muons on  $H_2$  (with respect to energy loss, scattering and their correlation) can be accurately determined using fundamental principles and available atomic and molecular data. We have used this double differential cross section to derive a database of probability maps for thin absorbers. We have then tracked muons through thicker absorbers by Monte Carlo. Results are substantially more consistent with available experimental data than previous calculations. Simulations of a long MICE cooling channel and a ring cooler, and a study of equilibrium normalised emittance, consistently confirm that ionisation cooling in liquid hydrogen is significantly more effective than has been suggested by earlier simulations using less rigorous physics.

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